EVALUATION OF THE SYSTEM-WIDE EFFECTS OF ON-LINE ESTIMATION METHODS ON FEEDBACK BASED COORDINATED RAMP METERING STRATEGY

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Abstract: The critical density of a freeway link is subject to changes over time owing to such circumstances as environmental conditions (snow, rain, etc.) and traffic incidents. Due to the critical density's impact on the performance of some ramp metering strategies that make use of it as a threshold value for control action, it is indispensable to trace the real value of the critical density. The focus of this paper is to evaluate the effects of the novel on-line critical density estimation methods proposed by Ozbay et al. (2006) on the performance of the two new feedback based coordinated ramp metering strategies, namely C-MIXCROS and D-MIXCROS (Kachroo and Ozbay 2003), that explicitly consider ramp queues using microscopic simulation models on a 11-mile corridor of I-295 in South Jersey under various demand conditions.

Keywords: Coordinated ramp metering, closed loop controller, nonlinear control, feedback control, Kalman filter, ramp queues, performance, total travel time.

1. INTRODUCTION AND MOTIVATION

The continuous increase of traffic demand has led to increasingly severe recurrent (occurring daily during rush hours) and non-recurrent (due to incidents) congested traffic conditions, both. One of the most efficient and direct control measures that are typically employed in freeway networks is ramp metering. Ramp metering improves freeway flow by breaking up platoons, and allowing more efficient merging, reduction of accidents, and fuel consumption. Freeway ramp control can be open loop (in general time-of day dependent) or closed loop (traffic responsive). In the first case, controls are derived from a priori known such traffic data as demands, occupancies, (e.g. Demand Capacity (Masher et al. 1975), while closed loop controls directly react to existing traffic conditions (e.g., ALINEA (Papageorgiou et al. 1991), MIXCROS (Kachroo and Ozbay 2003)).

Overall, there are two types of ramp metering. Local ramp metering considers an isolated section of the network consisting of a freeway section with one on-ramp, and the controller responds only to the changes in the local conditions. Coordinated ramp metering is the application of ramp metering to a series of entrance ramps with the goal of coordinating the response of all the ramps in the system. The disadvantage of local ramp metering is its lack of coordination between ramps in order to work toward optimization of the freeway facility. Coordinated ramp metering, on the other hand, can become very complex and expensive to implement and maintain.

Success of the on-line implementation of feedback-based ramp metering strategies such as "mixed-feedback-based" ramp control strategy, namely MIXCROS, and ALINEA depend on many factors, such as the regulator parameter (K_R) and the appropriate choice of meter locations. In addition to the need for careful calibration of the control parameters, it is important to improve the performance of the ramp control by introducing adaptive techniques that enable the control strategies to respond in real time to changes in control parameters such as jam density and free flow speed. One important factor can be the use of a set (or critical) occupancy (or density) value. This value is dependent on the capacity of the downstream freeway link, used to change the control action.

Capacity is the maximum hourly rate at which vehicles reasonably can be expected to traverse a point or a uniform section (HCM). In the HCM and other traffic engineering literature, the capacity of a freeway section is traditionally quantified using a constant (average) value. It is determined through observation of two of the primary parameters of traffic flow (flow and density). If it is possible to match a best-fit parabolic curve to the entire data set both for stable and unstable flows, the capacity can be estimated as the extreme point of this parabola. Smaragdis et al. (2004) argue that this procedure of critical occupancy determination may not be feasible or fully satisfactory under certain conditions. For instance, in the case where no ramp metering controls are implemented along the corridor, the occurrence of congestion on the upstream locations of the network can lead to reduced freeway traffic flow. Downstream locations of the network may never reach critical occupancy. However, these locations may later require metering as a result of increased traffic flow from the metered ramps located upstream along the corridor. In such a case, critical occupancy cannot be determined for these locations in No Control scenario. Furthermore, capacity is not a constant value. The stochastic nature of the capacity of a freeway link can be attributed to the variability of traffic characteristics. Changing traffic flow (including traffic conditions and driver behaviour), traffic composition, and "external" parameters such as the geometry and environmental conditions of the section all play a role. Adverse weather, for example, clearly affects both the flow-occupancy and speed-flow relationships. Therefore, maximum observed flows (at critical occupancy) usually decrease during adverse weather (Ibrahim and Hall 1994). When the critical occupancy is selected to be less than its real value, control laws tend to behave in such a conservative manner that they lead to excessive usage of on-ramp storage. This causes unwanted increases in on-ramp travel time. On the other hand, selecting higher occupancy thresholds results in higher volume and delay on the freeway, causing unwanted mainline delays. Thus, implementing ramp metering strategies using on-line estimation of critical density ensures better performance.

A strategy that automatically adapts to the real-time change of critical density, based on the freeway traffic conditions at the bottleneck locations, can prove efficient. Similar studies are reported in the literature (e.g., (Wang and Papageorgiou 2005), (Sun et al. 2003), (Sun et al. 2004), (Szeto et al. 1972), (Gazis and Liu 2003). In (Sun et al. 2003), a general approach to the real-time estimation of the complete traffic state in freeway stretches is developed based on the extended Kalman Filter (EKF). The EKF has also been applied in the past to obtain improved density estimates ((Sun et al. 2004), (Szeto et al. 1972)) by coupling the detector counts with independent density estimates, which are subject to uncorrelated errors. In Gazis and Liu (2003), the EKF is employed for estimating vehicle counts for two roadway sections in tandem.

Ozbay et al. (2006) propose two methods for the online estimation of critical density using EKF and KF; they take the critical density as the state variable to be determined through downstream freeway flow measurements. This new methodology is tested using both macroscopic (MATLAB) and microscopic (PARAMICS) simulation environments. Based on the simulation results (Ozbay et al. 2006), the proposed methods show promise by improving the system-wide performance of MIXCROS implementation on an isolated ramp system (e.g., by decreasing total travel time and increasing average speed). Although MIXCROS proves to be very effective in reducing the congestion on the isolated ramp system while keeping the on-ramp queue at an acceptable level, because it is a local feedback based ramp metering strategy, it does not always produce significant improvement at the system level when implemented on multiple ramps under varying traffic conditions (Ozbay et al. 2006). Thus, in this paper, the effect of online critical density estimation methods implemented as a part of ramp metering strategies is evaluated using two coordinated ramp control strategies, namely D-MIXCROS and C-MIXCROS. All the evaluations are conducted using PARAMICS.

THEORY

In this section, a brief description of the online critical density estimation methodology proposed in Ozbay et al. (2006) and the feedback based coordinated ramp metering strategies C-MIXCROS and D-MIXCROS are presented. However, the complete derivation and detailed evaluation of the online estimation methods and the control laws, which is outside the scope of this paper, are given in, Yasar et al. (2006), and Kachroo and Ozbay (2003), respectively.

The equations in this section are given for any merge segment of Figure 1. There has been a large amount of work on speed-density relationship in the past. In a recent study (Chen et al. 2004), it was reported that there are no theoretical models that are always consistent with the measured field data in reality. As to the empirical approaches, many kinds of traffic-stream curves have been reported and none are accepted widely. The current status of mathematical models for speed-flow concentration relationships is in a state of flux. However, each proposed model has different strengths and weaknesses. No replacement models have yet been developed (Hall 2005, Kerner 2002, Banks 1995). Hence, in this paper, for speed-density relationship, the fundamental diagram features are utilized.

The main idea behind the EKF-based estimation of critical density is to employ critical density directly as the state variable to be filtered using downstream freeway flow measurements. Let $\mathrm{k} = 1, 2, ...$ index time; $k = 1$ refers to the initial time. Let us assume that the process has a state parameter $x = \rho_{cr}$, which is governed by the stochastic difference equation:

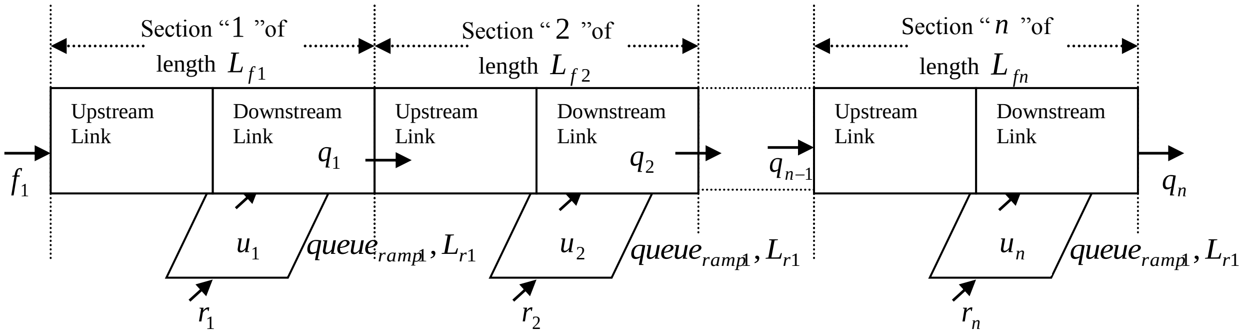

Fig.1. "n" freeway sections with one on-ramp.

$$\hat{x}(k) = A(t)\hat{x}(k-1) + \xi(k) \tag{1}$$

Here, for simplicity, it is further assumed that $\forall t, A(t) = 1$. The measurement model that describes the relationship between the state and the measurements is presented in the observation equation below:

$$\hat{q}_{out}(k) = v_f \rho(k)(1 - \frac{\rho(k)}{\hat{\rho}_{jam}}) + \theta(k) \tag{2}$$

where from the fundamental relationship $\hat{\rho}_{jam} = 2\hat{\rho}_{cr}$, v_f is the free flow speed, $\xi(k)$ and $\theta(k)$ are the system and output noise, respectively. It is assumed that ξ and θ have a zero mean and they are independent of each other, white, and with normal probability distributions. A nonlinear relationship (2) exists between the state and measurement variable. Therefore, EKF can be used, which applies the standard KF to nonlinear systems with additive white noise by continually updating a linearization (a linear Taylor expansion of the system; i.e., the first two terms of the approximation) around the previous state estimate, starting with an initial prediction. The new governing equation for the measurement that linearizes an estimate about the a priori estimate of the state (1) becomes:

$$q_{out}(k) \approx \hat{q}_{out}(\hat{x}^-(k)) + H(k)(\hat{x}(k) - \hat{x}^-(k)) + \Gamma\theta(k)$$

$H(k)$ is the partial derivative of measurement equation (2) with respect to state variable $\hat{x}$ at the a priori estimate $\hat{x}^-(k)$, and Γ is the partial derivative of the measurement equation with respect to $\theta(k)$, $p(\Gamma\theta(k)) \approx N(0, \Gamma\Theta(k)\Gamma^T)$. The state parameter can be replaced by a new term to provide a framework for the KF application for the on-line estimation of critical density, which eliminates the need for linearization of the measurement equation. This new state parameter is defined as $\hat{x}(k) = 1/\hat{\rho}_{cr}(k)$. It is assumed that the state changes according to (1). With the substitution of the new state parameter, the measurement model (2) yields a linear (albeit time varying) measurement equation, which is the framework for standard KF. Therefore, there is no need for an extended version whose results in general are not theoretically as complete as those of the standard ones (Welch and Bishop 2001). After defining state and measurement equations of the proposed methods, KF methodology (Gelb, A. et al. 1974) steps are followed for the on-line estimation of the critical density of a freeway link.

In Figure 1, n is the number of freeway sections, f_1 (veh/hr) is the flow entering the freeway at the first section, q_i is the flow leaving the freeway section i (veh/hr), r_i is the flow entering the ramp (veh/hr), u_i is the metered flow (veh/hr), ρ_i is the freeway density (veh/mi), $\rho_{c(i)}$ is the critical density (veh/mi), T is the time step duration (hr), $w_{1(i)}, w_{2(i)}$ are the weight factors ($w_{1(i)} + w_{2(i)} = 1$), $queue_{rampi}$ is the queue length on the ramp (veh/mi), K is the control gain ($0 < K < 1$), L_{fi} and L_{ri} are the length of the freeway and ramp section i (mi), respectively.

Coordinated version of MIXCROS (C-MIXCROS and D-MIXCROS) aims to maximize the throughput on all the freeway sections without creating long queues on all the metered ramps. The proportional-derivative state feedback control logic (4) and direct regulation of on-ramp queues are employed in the derivation of this proposed coordinated ramp metering strategy. This coordinated traffic-responsive ramp-metering control achieves its goal by minimizing the following error function. Therefore, control objective of coordinated MIXCROS is to make the following error function go to zero as the time, t, goes to infinity:

$$e(t) = \sum_{i=1}^{n} \left(\left| w_{1(i)} x_{1(i)}(t) \right| + \left| w_{2(i)} x_{2(i)}(t) \right| \right)_i \tag{3}$$

$x_{1(i)}(t) = \rho_i(t) - \rho_{cr(i)}$, $x_{2(i)}(t) = queue_{rampi}(t)$,

$i = 1,2,...n$ i : section index

The error function (3) is defined as summation of the absolute values of the state variables $x_{1(i)}$ and

$x_{2(i)}$. $x_{1(i)}$ is the state variable that represents the freeway section of the network, and $x_{2(i)}$ represents the on-ramp queue. (3) takes these state variables into account, determines how much importance should be given to freeway density and queue length on the ramp with the help of weights, w_1 and w_2. The control objective can be achieved by designing control law that makes the system follow the below closed loop dynamics:

$$\dot{e}(t) + Ke(t) = 0. \qquad (4)$$

The system can be in 2^n regions, where n is the number of freeway sections in the network due to the first term in the error function. These 2^n regions can be combined to come up with a control law that is applicable in all regions with the help of a function $sign$ such that

$$sign = \begin{cases} 1 & if\ \rho_i(t) > \rho_{cr(i)} \\ -1 & else \end{cases}.$$

The condition the control variables should satisfy is the following:

$$Gu(t) = -F(t) - Ke(t)$$

where,

$$F(t) = \sum_{i=1}^{n} \left(\begin{array}{l} w_{1(i)} \times sign \times \left(\frac{1}{L_{fi}} \times (-q_i(t) + f_i(t)) \right) \\ + w_{2(i)} \left(\frac{1}{L_{ri}} \times r_i(t) \right) \end{array} \right)$$

and

$$Gu(t) = \sum_{i=1}^{n} (sign \times w_{1(i)} \times \frac{1}{L_{fi}} - w_{2(i)} \times \frac{1}{L_{ri}}) \times u_i(t).$$

The control law can be designed in a decoupled or coupled way. The decoupled control law (D-MIXCROS) for each freeway section is:

$$u_i(t) = (sign \times w_{1(i)} \times \frac{1}{L_{fi}} - w_{2(i)} \times \frac{1}{L_{ri}})^{-1} \times (-F_i(t) - Ke_i(t)).$$

The coupled control law (C-MIXCROS) is:

$$u_i(t) = \alpha_i \times (sign \times w_{1(i)} \times \frac{1}{L_{fi}} - w_{2(i)} \times \frac{1}{L_{ri}})^{-1} \times (-F(t) - Ke(t))$$

where $\sum_{i=1}^{n} \alpha_i = 1$, $F = \sum_{i=1}^{n} F_i(t)$, and $e(t) = \sum_{i=1}^{n} e_i(t)$.

3. METHODOLOGY

C-MIXCROS and D-MIXCROS are implemented with and without proposed on-line estimation methods (using KF and EKF) and compared with No Control case for three demand scenarios (21 cases), whose congestion levels are listed in Table 1. Then, the performance of the proposed methods in the presence of disturbances, which are created by altering speed limits on the freeway links along the corridor, are observed (6 cases). The congestion level is the percent of the time that the downstream link occupancy is greater than the critical occupancy. All simulations are run for 3 hours with different seed values for the statistical analysis of the results, which ensured a 95% confidence level.

Table 1 Congestion Levels on Each Ramp

	1st Ramp	2nd Ramp	3rd Ramp	4th Ramp
1st Demand Level	27%	60%	68%	20%
2nd Demand Level	53%	39%	58%	9%
3rd Demand Level	0%	17%	35%	24%

PARAMICS model of the section of I-295 in South Jersey is created using the available geometric and traffic demand data (Figure 2). The calibrated and validated model of the 11 miles long 3-lane freeway section includes the junctions of I-295 with Route 38, State HWY 73, State HWY 70 and Berlin Rd. Each on-ramp has 1-lane. The speed limit on the freeway links is 60 mph An Application Programming Interface (API) is written to assign green times and to compute critical density in PARAMICS model of the network. In the API file, it is made sure that the calculated green phase duration is within specified limits, i.e. minimum and maximum values are 2 and 15 seconds respectively. The critical density is determined every minute. Statistics are collected from the detectors located downstream and upstream of the ramp and two additional detectors, one at the exit and one at the entrance of the ramp.

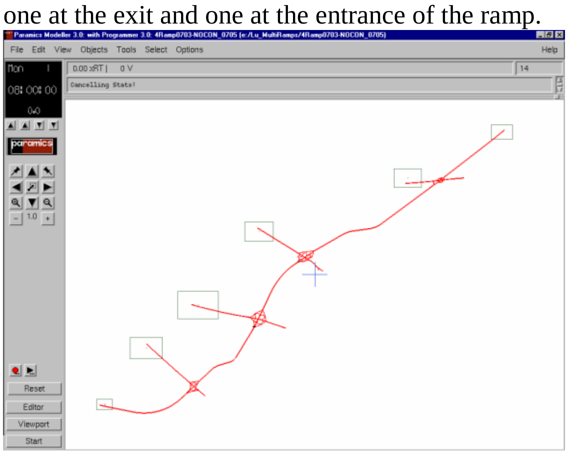

Fig. 2. Network with 4 on-ramps and 4 off-ramps.

4. SIMULATION RESULTS

Tables 2 and 3 summarize the main findings for three demand scenarios. The values in change rows are the percent changes with respect to the No Control scenario. Thus, improvements compared with the No Control are indicated by negative values for the average travel time (A, sec/veh) and the total travel time (C, veh.hr), and by positive values for the average speed (B, mph).

The average on-ramp queue lengths in all scenarios are approximately similar to the No Control case. Ramp metering controls seem to be more effective under certain demand patterns than others. As traffic demand increases, ramp metering tends to be more effective in reducing system travel time (Tables 2 and 3). The reason for reduced ramp metering performance for the third demand scenario is the low level of congestion on each ramp system. It is also claimed in other studies that the effectiveness of ramp control varies over the severity of congestion (Brilon et al. 2005, Zhang et al. 2001). Both C-MIXCROS and D-MIXCROS decrease the average travel time and increase the average speed regardless of the demand scenario compared with No Control. However, these improvements are comparatively less for the scenarios with constant critical density. For instance, average travel time is decreased by 6.45 % when C-MIXCROS is implemented using the proposed method with KF (2nd demand level), whereas the same ramp metering strategy is able to decrease the same measure by only 3.72 % with constant critical density (Table 3).

Table 2. System-Wide Results for D-MIXCROS

	Demand 1		Demand 2		Demand 3	
	A	B	A	B	A	B
No Control	646.8	55	713.3	49.6	616.5	57.8
D-MIXCROS	621.5	57.3	662.2	54.1	616.8	57.8
Change (%)	-3.92	4.1	-7.16	9.1	0.05	0.04
D-MIXCROS using EKF	622.8	57.3	661.9	54.2	611.8	58.1
Change (%)	-3.72	4.1	-7.21	9.3	-0.76	0.48
D-MIXCROS using KF	621.3	57.3	658.2	54.4	613.8	58.1
Change (%)	-3.94	4.2	-7.73	9.7	-0.43	0.43

Table 3. System-Wide Results for C-MIXCROS

	Demand 1		Demand 2		Demand 3	
	A	B	A	B	A	B
No Control	646.8	55	713.3	49.6	616.5	57.8
C-MIXCROS	622.7	57.3	686.8	52.2	613.0	58.1
Change (%)	-3.73	4.1	-3.72	5.14	-0.56	0.43
C-MIXCROS using EKF	626.2	56.8	673.8	53.1	610.5	58.3
Change (%)	-3.19	3.3	-5.54	7.01	-0.97	0.78
C-MIXCROS using KF	626.8	56.8	667.3	53.5	612.3	58.1
Change (%)	-3.09	3.2	-6.45	7.91	-0.68	0.56

Nevertheless, C-MIXCROS and D-MIXCROS with proposed methods (KF and EKF) made relatively small improvements when compared with the constant critical density implementation for the three demand levels tested (Tables 2 and 3).

Hence, in order to observe the effects of the proposed methods on the system-wide performance of C-MIXCROS and D-MIXCROS under the changing capacity conditions, the speed limits on the freeway links along the corridor are altered using API code. This type of speed reduction can be due to low visibility as result of fog or heavy rain. The simulation results are shown in Tables 4 and 5. C-MIXCROS and D-MIXCROS with constant critical density decrease the total travel time approximately 3 % compared with the No Control case. On the other hand, the proposed methods improve the performances of ramp metering implementations significantly. That is, the improvements obtained by the proposed methods become quite evident compared to the ramp controls with constant critical density utilization.

For instance, D-MIXCROS with the proposed on-line estimation method using EKF decreases the average travel time by 8.59 % while increasing the mean vehicle speed by 10.55 %. Alternatively, D-MIXCROS with constant critical density provides 2.97 % decrease in average travel time and 3.46 % increase in average speed.

Table 4. System-Wide Results for C-MIXCROS

	A	B	C
No Control	922.63	37.6	5285
D-MIXCROS	895.27	38.9	5138
Change (%)	-2.97	3.46	-2.77
D-MIXCROS_EKF	812.70	43.6	4649
Change (%)	-11.92	16.05	-12.03
D-MIXCROS_KF	843.37	41.6	4828
Change (%)	-8.59	10.55	-8.65

Table 5. System-Wide Results for C-MIXCROS

	A	B	C
No Control	922.63	37.6	5285
C-MIXCROS	903.03	38.4	5188
Change (%)	-2.12	2.13	-1.82
C-MIXCROS_EKF	833.33	42.1	4753
Change (%)	-9.68	11.97	-10.05
C-MIXCROS_KF	856.50	40.3	4970
Change (%)	-7.17	7.18	-5.95

5.CONCLUSIONS

On-line parameter estimation techniques proposed by Ozbay et al. (2006) combined with two new feedback based coordinated ramp controls, namely C-MIXCROS and D-MIXCROS (Kachroo and Ozbay 2003) are evaluated using microscopic simulation models on a 11-mile corridor of I-295 in South Jersey under a variety of demand conditions. Feedback based coordinated ramp metering controls C-MIXCROS and D-MIXCROS with constant critical density are able to provide system-wide improvements in the PARAMICS model of the test network. However, these improvements are even greater after the implementation of the proposed on-line estimation methods.

It is demonstrated that the proposed on-line critical density estimation methods are valuable in terms of providing large system-wide benefits when the disturbances in the traffic conditions, traffic composition, and the external parameters such as the geometry and environmental conditions of the section are present. The simulation results show that well-calibrated C-MIXCROS and D-MIXCROS with on-line estimation of critical density (with KF and EKF) generate up to 12 % total travel time savings and 16 % average speed increase compared with the No Control scenario over the 3-hr peak period.

Microscopic simulation results of both methods using KF and EKF show approximately the same level of system-wide improvements in the form of increased average speed and reduced total travel time. Both results are superior to the results collected using constant critical density over the course of the simulation. However, KF is a more desirable approach because it is a linear filter that offers well-known convergence and optimality results.

REFERENCES

Banks, J.H. (1995). Another Look at Priori Relationship Among Traffic Flow Characteristics. *Transportation Research Record*, **No. 1510**, TRB, Washington D.C., pp. 1-10.

Brilon, W., J. Geistefeldt, and M. Regler (2005). Reliability of Freeway Traffic Flow: A Stochastic Concept of Capacity. Transportation and Traffic Theory Flow, Dynamics and Human Interaction. *Proceedings of the 16th International Symposium on Transportation and Traffic Theory*, University of Maryland, College Park, Maryland.

Chen et al. (2004). Freeway Traffic Stream Modeling Based on Principal Curves and It's Analysis. *IEEE Transaction on Intelligent Transportation Systems*, **Vol. 5**, **No. 4**, pp. 246-258.

Gazis, D., and C. Liu (2003). Kalman Filtering Estimation of Traffic Counts for Two Network Links in Tandem. *Transportation Research Part B* **37**, pp. 737-745.

Gelb, A. et al. (1974). Applied Optimal Estimation, The Analytic Sciences Corporation, pp.188.

Hall, F. Traffic Flow Theory A State of the Art Report Traffic Stream Characteristics by Fred L. Hall, Revised Monograph on Traffic Flow Theory, U.S. Department of Transportation, Federal Highway Administration, http://www.tfhrc.gov/its/tft/tft.htm

Highway Capacity Manual (2000).

Ibrahim, A. T., and F. L. Hall (1994). Effect of Adverse Weather Conditions on Speed-Flow-Occupancy Relationships. *Transportation Research Record*, **No. 1457**, TRB, National Research Council, Washington, D.C.

Kachroo, P. and K. Ozbay (2003). *Feedback Ramp Metering in Intelligent Transportation Systems*. Kluwer Academics.

Kerner, B.S. (2002). Theory of Congested Highway Traffic: Empirical Features and Methods of Tracing and Prediction. *Transportation and Traffic Theory in the 21st Century*, M. A. P. Taylor, Ed. Amsterdam, The Netherlands: Elsevier, pp. 417-439.

Masher, D. P., et al. (1975). Guidelines for Design and Operating of Ramp Control Systems. *Stanford Research Institute Report* NCHRP 3-22, SRI Project 3340, SRI, Menid Park, CA.

Ozbay, K., I. Yasar, and P. Kachroo (2004). Comprehensive Evaluation of Feedback Based Freeway Ramp Metering Strategy by Using Microscopic Simulation: Taking Ramp Queues into Account. *Transportation Research Record*, **No. 1867**, TRB, National Research Council, Washington, D.C., pp. 89-96.

Ozbay, K., I. Yasar, and P. Kachroo (2005). Evaluation of Application of Extended Kalman Filtering to On-line Estimation of Critical Density, Working Paper, Rutgers Intelligent Transportation Systems (RITS) Laboratory.

Ozbay, K., I. Yasar and P. Kachroo (2006). Development and Evaluation of On-Line Estimation Methods for Feedback Based Freeway Ramp Metering Strategy. Presented in 85th TRB Annual Meeting.

Papageorgiou, M., H. S. Habib, and J. M. Blosseville (1991). ALINEA: A Local Feedback Control Law for On-ramp Metering. *Transportation Research Record*, **No. 1320**, TRB, Washington, D.C., pp. 58-64.

Smaragdis, E., M. Papageorgiou, and E. Kosmatopoulos (2004). A Flow–Maximizing Adaptive Local Ramp Metering Strategy. *Transportation Research Part B* **38**, pp. 251-270.

Sun, X., L. Munoz, and R. Horowitz (2003). Highway Traffic State Estimation Using Improved Mixture Kalman Filters for Effective Ramp Metering Control. *Proceedings of the 42nd IEEE Conference on Decision and Control*. Maui, Hawaii, USA, pp. 6333-6338.

Sun, X., L. Munoz, and R. Horowitz (2004). Mixture Kalman Filter Based Highway Congestion Mode and Vehicle Density Estimator and its Application. *Proceeding of the 2004 American Control Conference*, Boston, Massachusetts, pp. 2098-2103.

Szeto, M. W., and D. C. Gazis (1972). Application of Kalman Filtering to the Surveillance and Control of Traffic Systems. *Transportation Science* **Vol. 6 (4)**, pp. 419-439.

Wang, Y. and M. Papageorgiou (2005). Real-time Freeway Traffic State Estimation Based on Extended Kalman Filter: A General Approach. *Transportation Research Part B* **39**, pp. 141-167.

Welch, D. C. and G. Bishop (2001). *An Introduction to Kalman Filter*, SIGGRAPH 2001-Course 8.

Yasar, I., K. Ozbay and P. Kachroo (2006). Evaluation of New Coordinated Feedback-Based Freeway Ramp-Metering Strategy using Microscopic and Macroscopic Simulation Models, *9th International Conference on Applications of Advanced Technology in Transportation, AATT*, Chicago, Illinois USA.

Yin, Y., Liu, H., and Benouar, H. (2004). A Note on Equity of Ramp Metering. *IEEE Intelligent Transportation Systems Conference*, Washington, D. C., USA.

Zhang, H. M. and W. W. Recker (1999). On Optimal Freeway Ramp Control Policies for Congested Traffic Corridors. *Transportation Research, Part B: Methodological*, **33, 6**, p. 417-436.

Zhang et al. (2001). Evaluation of On-ramp Control Algorithms. *California PATH Research Report*,UCB-ITS-PRR-2001-36.