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Improved Online Estimation Methods for a Feedback-Based Freeway Ramp Metering Strategy

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Abstract--**The critical density of a freeway link is subject to changes over time owing to such circumstances as environmental conditions (snow, rain, etc.) and traffic incidents. Because of the critical density impacts on the performance of some ramp metering strategies that make use of it as a threshold value for control action, it is necessary to trace the real value of critical density. This paper presents improvements to the methodology for the online estimation of critical density using the extended Kalman filter (EKF) proposed by Ozbay et al. (2006) [1]. Basically, critical density and density of the freeway section are chosen as the state variables to be determined using the system output, namely the measurement of traffic flow and occupancy on the downstream freeway link. The effectiveness of the proposed method is evaluated using the feedback-based ramp metering strategy ALINEA [2]. A number of simulations are run to investigate the sensitivity of the proposed methodology with respect to initial estimates and time step size selection. Also, the methodology's capability of tracking gradual and sudden changes in real-time critical density is examined. This new methodology provided successful performances based on the macroscopic simulation evaluation using MATLAB.**

I. INTRODUCTION AND MOTIVATION

Ramp metering is one of the most beneficial management strategies in alleviating congestion in freeway networks. Success of the online implementation of feedback-based ramp metering strategies such as the "mixed-feedback-based" ramp control strategy, namely MIXCROS [3], and ALINEA [2] depend on many factors, including careful calibration of the control parameters and the appropriate choice of meter locations.

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One important factor is the use of a set (or critical) occupancy (or density) value. This value is dependent on the capacity of the downstream freeway link, which is used to change the control action. Capacity is the maximum hourly rate at which vehicles reasonably can be expected to traverse a point or a uniform section [4]. In ramp metering implementations, the capacity of the downstream freeway link has been treated mostly as a constant parameter. If it is possible to match a best-fit parabolic curve to the entire flow density data set both for stable and unstable flows, the capacity can be estimated as the extreme point of this parabola. This way of off-line parameter estimation is a cumbersome procedure. Also, it is argued by Smaragdis et al. (2004) [5] that this procedure may not be feasible or fully satisfactory under certain conditions. For instance, if no ramp metering controls are implemented along the corridor, the occurrence of congestion on the upstream locations of the network can lead to reduced freeway traffic flow. Downstream locations of the network may never reach critical occupancy. However, these locations may later require metering as a result of increased traffic flow from the metered ramps located upstream along the corridor. In such a case, critical occupancy cannot be determined for these locations in the No Control scenario. Furthermore, capacity is not a constant value. The stochastic nature of the capacity of a freeway link can be attributed to the variability of traffic characteristics. Changing traffic flow (including traffic conditions and driver behavior), traffic composition, and "external" parameters such as the geometry and environmental conditions of the section all play a role. Adverse weather, for example, clearly affects both the flow–occupancy and speed–flow relationships. Therefore, maximum observed flows (at critical occupancy) usually decrease during adverse weather [6]. When the critical occupancy is selected to be less than its real value, control laws tend to behave in such a conservative manner that they lead to excessive usage of on-ramp storage. This causes unwanted increases in on-ramp travel time. On the other hand, selecting higher occupancy thresholds results in higher volume and delay on the freeway, causing unwanted mainline delays. Ultimately, real-time information on changes in the critical parameters that are used in ramp metering strategies is vital to achieving efficient traffic management. Therefore, the aim of this study is to develop a strategy that automatically adapts to the real-time change of critical density, based on the freeway traffic conditions at the bottleneck locations.

Traffic-responsive ramp metering strategies rely on such real-time traffic data as volume and occupancy for the algorithm calculations. These data are readily available from detectors along the freeway segments. The proposed methodologies, namely A-1 and A-2, use these data to filter two real-time traffic states, namely critical density and density of the freeway section (Figure 1). A-1 and A-2 are employed in ramp metering control law equations for changing the control action. The idea of estimating freeway traffic states in real-time based on Kalman Filter may not be new. However, this paper proposes an improvement to the previous online critical density estimation methods by Ozbay et al. (2006) [1].

The methods rely on (1) a model based on the principle of the conservation of vehicles and (2) a fundamental diagram describing the speed–density relationship. Evaluation of the methods is performed in a macroscopic simulation environment under different scenarios with varying demands.

Similar studies are reported in the literature ([7], [8], [9], [10] and [11]). In [8], a general approach to the real-time estimation of the complete traffic state in freeway stretches is developed based on the extended Kalman Filter (EKF). The EKF has also been applied in the past to obtain improved density estimates ([9], [10]) by coupling the detector counts with independent density estimates, which are subject to uncorrelated errors. In [11], the EKF is employed for estimating vehicle counts for two roadway sections in tandem. [1] and [5] develop strategies that allow for the automatic tracking of the critical occupancy whenever it cannot be estimated beforehand or whenever it is subject to real-time change as a result of environmental conditions or traffic composition.

II. TRAFFIC FLOW MODELING

Figure 1 shows a schematic diagram of a freeway section with one on-ramp. The discretized conservation of vehicles principle can be stated as:

$$\rho(k) = \rho(k-1) + \frac{\Delta T}{\Delta x}(q_{in}(k) - q_{out}(k) + u(k)). \quad (1)$$

The nonlinear relationship between critical density ($\rho_{cr} = \rho_{jam}/2$) and downstream freeway flow q_{out} ($q_{out} = \rho v$, the fundamental relationship for traffic flow) is obtained as:

$$q_{out} = v_f \rho (1 - \frac{\rho}{\rho_{jam}}), \quad (2)$$

where v_f is the free flow speed and ρ_{jam} is the jam density.

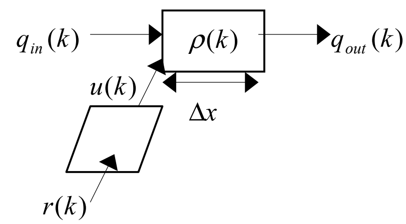

Fig 1. Ramp system.

Two of the measurements that have traditionally been used to monitor traffic operations on freeways include occupancy and volume. Occupancy has a direct relationship to density:

$$\rho = \frac{5280 \times occ}{(\overline{L}_V + \overline{L}_D)} \quad (3)$$

occ is the occupancy, $\overline{L}_V$ (ft) is the average vehicle length and $\overline{L}_D$ (ft) is the sensitivity of the sensor [12].

These available real-time measurements (i.e., occupancy and flow data) can be used to filter two state variables, namely density and critical density, for the implementation of ramp metering strategies.

III. ONLINE ESTIMATION METHODS: APPROACH 1 AND APPROACH 2

APPROACH 1 (A-1): The main idea behind the improved EKF-based estimation of critical density is to employ critical density and density of the freeway section as the state variables to be filtered using downstream freeway flow measurements:

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} \rho(k) \\ inv\_\rho_{crit}(k) \end{bmatrix} \quad (4)$$

$$y(k) = [y_1(k)] = [q_{out}(k)]$$

APPROACH 2 (A-2): A-2 utilizes occupancy data, in addition to the real-time downstream flow measurements from the detectors located on the freeway, in order to filter critical density and density of the freeway section.

State and measurement variables of A-2 are:

$$x(k)=\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}=\begin{bmatrix} \rho(k) \\ inv\_\rho_{crit}(k) \end{bmatrix}, \quad (5)$$

$$y(k)=\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix}=\begin{bmatrix} q_{out}(k) \\ occ(k) \end{bmatrix}.$$

Let $k=1,2,...$ index time; $k=1$ refers to the initial time and $inv\_\hat{\rho}_{cr}(k)=1/\hat{\rho}_{cr}(k)$.

IV. FORMULATION OF THE METHODS: A-1 AND A-2

Let us assume that the process has state parameters x_1 and x_2 that are governed by the stochastic difference equations:

$$\hat{x}_1(k)=f(\hat{x}_1(k-1),\hat{x}_2(k-1),\xi_1(k-1)),$$

$$\hat{x}_2(k)=f(\hat{x}_2(k-1),\xi_2(k-1)),$$

with measurement equations:

$$\hat{y}_1(k)=f(\hat{x}_1(k),\hat{x}_2(k),\theta_1(k)),$$

$$\hat{y}_2(k)=f(\hat{x}_1(k),\theta_2(k)).$$

It is assumed that the states change according to the stochastic difference equations:

$$\hat{x}_1(k)=\hat{x}_1(k-1)+\frac{\Delta T}{\Delta x}(q_{in}(k-1) - \hat{x}_1(k-1)V_f(1-\hat{x}_1(k-1)\hat{x}_2(k-1)/2) + u(k-1))+\xi_1(k) \quad (4)$$

$$\hat{x}_2(k)=A(t)\hat{x}_2(k-1)+\xi_2(k) \quad (5)$$

Here, for simplicity, it is further assumed that $A(t)=1,\ \forall t=1$. The measurement models that describe the relationship between the state and the measurements are presented in the observation equations below:

$$\hat{y}_1(k)=\hat{x}_1(k)V_f(1-\hat{x}_1(k)\hat{x}_2(k)/2)+\theta_1(k), \quad (6)$$

$$y_2(k)=\frac{(\bar{L}_d+\bar{L}_v)}{5280}x_1(k)+\theta_2(k),$$

where $\xi_1(k)$, $\xi_2(k)$ and $\theta_1(k),\theta_2(k)$ are the system and output noises, respectively, whose covariance matrices determine the tracking properties of the resulting EKF [13].

The a priori estimate at step k is defined as the estimate made with knowledge of the process prior to step k and is denoted by a superscript minus sign. In other words, $x_1^-(k)$ stands for the a priori estimate of $x_1(k)$.

Because the first state and measurement equations x_1 and y_1 are nonlinear functions of the state parameters, the linearization method of Taylor expansion of the equations is used, leading to the extended Kalman Filter (EKF) method. The EKF approach is to apply the standard Kalman Filter (for linear systems) to nonlinear systems with additive white noise by continually updating a linearization around the previous state estimate, starting with an initial guess. This approach gives a simple and efficient algorithm to handle a nonlinear model. However, convergence to a reasonable estimate may not be obtained if the initial guess is poor or if the disturbances are so large that the linearization is inadequate to describe the system. The new filter equations for A-1 and A-2 can be expressed as:

A-1:

$$\hat{x}^-(k)=A\hat{x}(k-1)+d(k-1)+\xi(k), \quad (7)$$

$$\hat{y}^-(k)=H_1(k)\hat{x}^-(k)+g_1(k)+\theta_1(k); \quad (8)$$

A-2:

$$\hat{x}^-(k)=A\hat{x}(k-1)+d(k-1)+\xi(k), \quad (9)$$

$$\hat{y}^-(k)=H_2(k)\hat{x}^-(k)+g_2(k)+\theta_2(k); \quad (10)$$

where

$$A=\begin{bmatrix} (1-\frac{\Delta T}{\Delta x}V_f+\frac{\Delta T}{\Delta x}V_f\hat{x}_1^-(k-1)\hat{x}_2^-(k-1)) & \frac{\Delta T}{2\Delta x}V_f(\hat{x}_1^-(k-1))^2 \\ 0 & 1 \end{bmatrix},$$

$$d=\begin{bmatrix} \frac{\Delta T}{\Delta x}q_{in}(k-1)+\frac{\Delta T}{\Delta x}u(k-1)-\frac{\Delta T}{\Delta x}V_f(\hat{x}_1^-(k-1))^2\hat{x}_2^-(k-1) \\ 0 \end{bmatrix},$$

$$H_1=\begin{bmatrix} (V_f-V_f\hat{x}_1(k-1)\hat{x}_2(k-1)) & -\frac{V_f}{2}(\hat{x}_1(k-1))^2 \end{bmatrix},$$

$$g_1=V_f(\hat{x}_1(k-1))^2\hat{x}_2(k-1),$$

$$H_2=\begin{bmatrix} (V_f-V_f\hat{x}_1(k-1)\hat{x}_2(k-1)) & -\frac{V_f}{2}(\hat{x}_1(k-1))^2 \\ \frac{(\bar{L}_d+\bar{L}_v)}{5280} & 0 \end{bmatrix},$$

$$g_2=\begin{bmatrix} V_f(\hat{x}_1(k-1))^2\hat{x}_2(k-1) \\ 0 \end{bmatrix}.$$

It is assumed that ξ_1,ξ_2 and $\theta_1(k),\theta_2(k)$ have a zero mean and that they are independent of each other, white, and with normal probability distributions:

$$p\{\xi_1(k)\}\approx N(0,\Xi_1(k)), p\{\xi_2(k)\}\approx N(0,\Xi_2(k)),$$

$$p\{\theta_1(k)\}\approx N(0,\Theta_1(k)), p\{\theta_2(k)\}\approx N(0,\Theta_2(k)),$$

where Ξ_1 and Ξ_2 are the process noise covariances and Θ_1 and Θ_2 are the measurement noise covariances.

Of course, one does not know the individual values of the noises ξ_1,ξ_2 and θ_1,θ_2 at each time step. However, one can approximate the state measurement without ξ_1,ξ_2 and θ_1,θ_2, respectively, as:

$$\hat{x}^-(k) \cong f(\hat{x}(k-1),0), \quad (11)$$

$$\hat{y}^-(k) \cong f(\hat{x}^-(k),0). \quad (12)$$

$P(k)$ and $P^-(k)$ are the a posteriori and a priori state estimation error covariance matrices, respectively, such that:

$$P(k) = E\{(x(k)-\hat{x}(k))(x(k)-\hat{x}(k))'\}, \quad (13)$$

$$P^-(k) = E\{(x(k)-\hat{x}^-(k))(x(k)-\hat{x}^-(k))'\}, \quad (14)$$

where $x(k)$ is the actual state parameter.

V. ESTIMATION ALGORITHM FOR CRITICAL DENSITY USING EXTENDED KALMAN FILTER: A-1 AND A-2

The EKF methodology for online estimation of the critical density of a freeway link is given as follows [13]:

1. Assume that the initial state $\hat{x}(0)$ is a random variable with known mean and covariance matrix P_0. Initialize the filter by setting the state variable (x) and the covariance matrix, namely

   $\hat{x}(0) = x_0$, $P(0) = P_0$.

2. Predict the state vector and the covariance matrix using the approximate values from (11) and (14), respectively

   $$\hat{x}^-(k) \cong f(\hat{x}(k-1),0),$$

   $$P^-(k) \cong AP(k)A^T + \Xi(k),$$

where $P^-(k)$ is the a priori state estimation error covariance.

3. Compute the Kalman gain matrix $K(k)$:

$$K(k) = P^-(k)H^T(k)(H(k)P^-(k)H^T(k)+\Theta(k))^{-1}$$

4. Update the state variable and its covariance matrix

$$\hat{x}(k) = \hat{x}^-(k) + K(k)(y(k)-\hat{y}^-(k))$$

where $y(k)$ is the actual measurement and $\hat{y}^-(k)$ is the approximate estimation, which can be determined using (8) or (10). Then, update the covariance matrix of the state variable using:

$$P(k) = (1-K(k)H(k))P^-(k).$$

4. Go to step 2 and iterate the algorithm when the updated real-time information is available.

VI. SIMULATION

The methods are implemented in the feedback-based ramp metering strategy ALINEA. ALINEA does not directly consider on-ramp queue. This is therefore handled through overriding restrictive metering rates where the metering rate is set to maximum when the on-ramp queue reaches a predetermined level. In order to concentrate only on the mainstream impact of the ramp metering strategy, ramp demand is kept at low values and no queue overriding tactics are used in each scenario tested. The regulator parameter K_R is chosen to be 240 veh/hr. Also, 90% of the critical occupancy is used as the set (desired) occupancy for all the cases.

Simulation is conducted on a ramp system (Figure 1) consisting of a one-lane (1 mile) freeway link and a one-lane (0.5 mile) ramp link. The mainstream capacity is approximately 1,500 veh/hr/lane and is obtained at density values in the range of 35 to 50 veh/mile/lane. The time discretization step for the ramp metering implementation equals 1 min. The simulation duration for each case is 5 hr.

The proposed methodology implementation requires the initial critical density estimate, which can be simply determined from (q_{out}, o_{out}) diagrams.

VII. SIMULATION RESULTS

The methods' tracking performances are assessed under visually clear traffic conditions (deterministic demands and ρ_{cr}) and in a noisy environment where stochastic noise is added to demand and ρ_{cr}. The methods' sensitivities to initial estimates, time step size, and gradual and sudden changes in the capacity of the freeway downstream link (Figure 1) are investigated using a MATLAB-based macroscopic simulation environment. The adaptation works properly for all the time step sizes and initial values tested (Figures 2a-2d). Both A-1 and A-2 are not sensitive to the selection of these parameters. A-1 traces the real ρ_{cr} more rapidly compared with A-2. However, both methods are quite successful in online estimation of the real ρ_{cr} under stochastic and deterministic conditions. Also, the methods have good adaptive capabilities for tracing gradual and sudden changes of critical occupancy (Figures2e-2h).

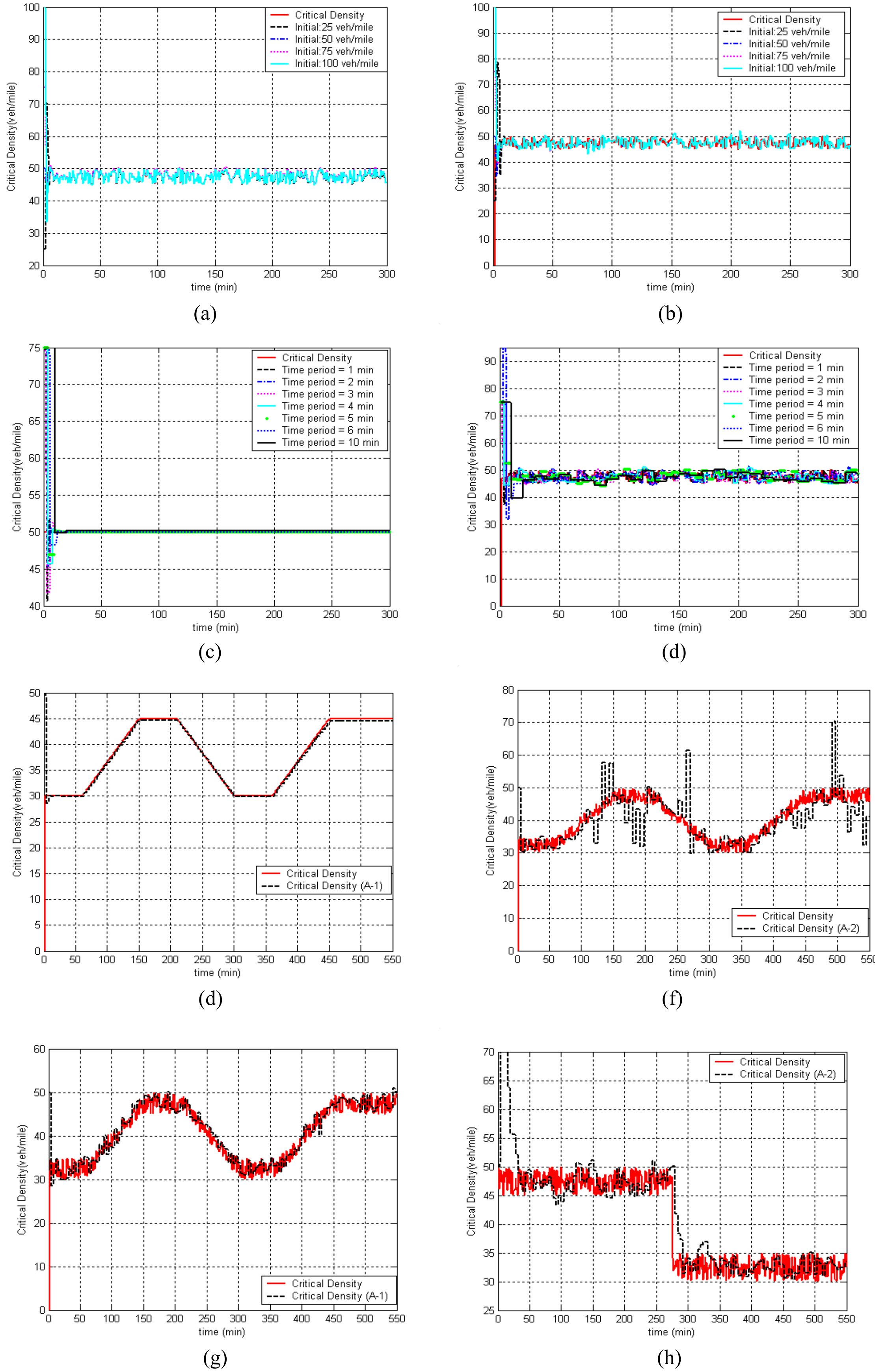

Fig 2. Tracking behaviors of A-1 (2a, 2c, 2e, and 2g) and A-2 (2b, 2d, 2f, and 2h).

VIII. CONCLUSIONS

Two new methods are proposed for the online estimation of critical density using EKF. A-1 takes the critical density and the density as the state variables to be determined through downstream freeway flow measurements. A-2 takes the critical density and the density as the state variables to be determined through downstream freeway flow and occupancy measurements. These new methodologies are tested using a macroscopic simulation environment (MATLAB).

The proposed online estimation methods have very simple algorithms. They efficiently track change in critical density, which can result from environmental conditions, traffic incidents, or stochastic fluctuations. The only parameters needed to implement this online estimation methodology are initial estimates of the critical density and the covariance matrix of the initial state estimate. The former can be simply determined from the flow density plots. The latter is a random variable with a known mean (Section IV). The time step size of the calculations can be assigned any increment of the metering update time, considering that the larger the time step sizes are, the slower the method converges to the real value of the critical density.

Evaluation of the proposed methods shows that these methods are not sensitive to the selection of the initial estimate or the time step size employed to estimate the new critical density. In other words, choice of the initial estimate and time step size did not affect the adaptive behavior of the methods. Hence, regarding parameter sensitivity, the proposed methods are quite robust (Ozbay et al., working paper, 2006). Another strength of the proposed methods is that they respond to sudden capacity changes very well. This can be very valuable during the peak period, when traffic conditions can change unexpectedly ([1], and Figure 2).

This paper demonstrates that the proposed improved methodologies for the online critical density estimation can be applied effectively during ramp metering implementations. The newly proposed methodologies are able to provide excellent real-time tracking of critical density on a freeway section to be utilized as the threshold in ramp metering strategies. Further study that involves microscopic testing in large networks is needed to understand the real-time behavior of the proposed methods.

REFERENCES

[1] Ozbay, K., I. Yasar, and P. Kachroo. “Development and Evaluation of On-Line Estimation Methods for Feedback Based Freeway Ramp Metering Strategy.” Presented in 85th TRB Annual Meeting, January 11-15, 2006.

[2] Papageorgiou, M., H. S. Habib, and J. M. Blosseville. “ALINEA: A Local Feedback Control Law for On-ramp Metering.” *Transportation Research Record* 1320, pp. 58-64, 1991.

[3] Kachroo, P. and K. Ozbay. *Feedback Ramp Metering in Intelligent Transportation Systems*, Kluwer Academics, 2003.

[4] Highway Capacity Manual (2000).

[5] Smaragdis, E., M. Papageorgiou, and E. Kosmatopoulos. “A Flow-Maximizing Adaptive Local Ramp Metering Strategy.” *Transportation Research Part B* 38, pp. 251-270, 2004.

[6] Ibrahim, A. T., and F. L. Hall. “Effect of Adverse Weather Conditions on Speed-Flow-Occupancy Relationships,” *Transportation Research Record* 1457, 1994.

[7] Wang, Y., and M. Papageorgiou. “Real-time Freeway Traffic State Estimation Based on Extended Kalman Filter: A General Approach.” *Transportation Research Part B* 39, pp. 141-167, 2005.

[8] Sun, X., L. Munoz, and R. Horowitz. “Highway Traffic State Estimation Using Improved Mixture Kalman Filters for Effective Ramp Metering Control.” *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, Hawaii, USA, pp. 6333-6338, 2003.

[9] Sun, X., L. Munoz, and R. Horowitz. “Mixture Kalman Filter Based Highway Congestion Mode and Vehicle Density Estimator and its Application.” *Proceeding of the 2004 American Control Conference*, Boston, Massachusetts, pp. 2098-2103, 2004.

[10] Szeto, M. W., and D. C. Gazis. “Application of Kalman Filtering to the Surveillance and Control of Traffic Systems.” *Transportation Science,* Vol. 6(4), pp. 419-439, 1972.

[11] Gazis, D., and C. Liu. “Kalman Filtering Estimation of Traffic Counts for Two Network Links in Tandem.” *Transportation Research Part B* 37, pp. 737-745, 2003.

[12] Mn/DOT Traffic Data,Density Calculation from Dectector Data, < http://data.dot.state.mn.us/datatools/Density.html >, March 14, 2006.

[13] Welch, D. C., and G. Bishop. *An Introduction to Kalman Filter*, SIGGRAPH 2001-Course 8, 2001.

[14] Gelb, A. et al.. Applied Optimal Estimation, The Analytic Sciences Corporation, pp.188, 1974.

[15] Kerner, B.S. “Theory of Congested Highway Traffic: Empirical Features and Methods of Tracing and Prediction.” *Transportation and Traffic Theory in the 21st Century*, M. A. P. Taylor, Ed. Amsterdam, The Netherlands: Elsevier, pp. 417-439, 2002.

[16] Brilon, W., J. Geistefeldt, and M. Regler. “Reliability of Freeway Traffic Flow: A Stochastic Concept of Capacity. Transportation and Traffic Theory Flow, Dynamics and Human Interaction.” *Proceedings of the 16th International Symposium on Transportation and Traffic Theory*, University of Maryland, College Park, Maryland, 2005.